Effects of the Rashba and the Dresselhaus Spin-orbit Interactions on the Quantum Transport and Spin Filtering in a Three-terminal Quantum Ring

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Abstract
The spin resolved conductance in a quantum ring with one input and two output leads in the presence of the Rashba spin-orbit interaction (RSOI) and the Dresselhaus spin-orbit interaction (DSOI) has been studied. The conditions needed for perfect spin polarization including the value of the Rashba and Dresselhaus coupling strength and the values of the electron energy are investigated coupled strongly with the leads. Our calculations are performed using the non-equilibrium Green’s function method and Griffith boundary condition in the framework of the tight binding model. It is shown that the spin-polarized transport and polarizability can be controlled by the RSOI and/or DSOI in the effects of applied magnetic field treading the ring, but also on the bias applied between the input and output leads. Results of this paper can be used in designing perfect spin inverters. The effects of relative positions of the drain electrodes on the perfect spin polarization are also investigated.

Keywords: three-terminal quantum ring; Rashba; Dresselhaus.

Introduction
During last few decades, there has been a lot of efforts, both theoretically an experimentally, to make electronic devices as small as possible. After invention of the spin field transistor in designing a spin field effect transistor by Datta and Das [1], there has been a growing interest in a new branch of physics named as Spintronics or spin-based electronics [2–3]. Unlike electronic devices which are based on charge of the electrons, Spintronics devices are based on spin of the electrons. One of major goals of Spintronics is generation of spin-polarized currents from an unpolarized charge current and their appropriate manipulation in a controllable environment. An effective way to attain this goal is due to interaction between the spin of the electron and orbital degree of freedom and magnetic properties. A good example to study such kind of effects is a
mesoscopic system such as quantum rings formed at the interface of semiconductor heterojunction structure, typically have size scales between the nanometer and few micrometers. In these systems have dimensions smaller or comparable to the phase coherent length and electrons have high mobility and long spin coherent time, so they support several quantum interference effects which can be controlled by external electric and magnetic fields. In a nutshell, mesoscopic rings provide a good platform for Spintronics application. In these mesoscopic systems, there are two different ways of creating and changing spin polarization: Zeeman interaction and spin orbit interaction (SOI) [4]. These studies are focused on the role of spin orbit interaction (SOI) in nonmagnetic device patterned in a two-dimensional electrons gas (2DEG). In general, there are two different source of electric field causing SOI in solid state systems: impurities in the conduction layer and lack of inversion symmetry. The SOI due to the impurities is usually very weak in epitaxially grown semiconductor quantum wells and can be neglected in practice. Therefore, semiconductor mesoscopic rings are ideal candidates for investigating SOI effects, so they are promising building blocks for designing nanodevices in electronic and spintronic engineering and have promising applications in design of spin-based digital logic devices.

During the past few years great efforts have been devoted to overcome the fundamental obstacles in the realization of spintronic devices, such as semiconductor mesoscopic ring for investigating SOI effects and the filtering of pure spin-polarization current. There are two types of SOIs, which are relevant for semiconductor spintronics. In 1995, Rashba first introduced the SO coupling [5], this kind of SOI is caused by the structure inversion asymmetry of a 2-DEG system. The strength of RSOI depends on the crystal composition in the quantum well and it is the largest for narrow gap semiconductor which makes it often an excellent approximation to neglect the contribution due to impurities and the other mechanism (DSOI). On the other hand, SOI created by structural inversion asymmetry (SIA) electrons moving through a lattice and the structure of which dose did not have inversion symmetry, feels an asymmetric crystal potential and the resulting spin-splitting of the conduction band was demonstrated analytically by Dresselhaus. Therefore, this kind of SOI which exist in zinc blended structure and due to SIA is known as Dresselhaus SO (DSOI) [6]. Therefore, the competition between these SOI mechanisms leads, in general, to complex behavior of the spin dynamics. In the last few years, the persistent current flowing in normal metal rings threaded by magnetic flux without leads was also considered by Loss and Goldbart in 1991 [7]. Nitta et al. proposed spin interference devices in 2003. This device was a one-dimensional ring connected with two conductor leads [8]. In 2008 Kalman et al. introduced another one-dimensional Spintronic with three-terminal quantum ring [9]. The effects of Rashba spin–orbit and magnetic flux on charge and spin currents in a quantum ring with three leads, has been investigated using the S-matrix method by Shelykh et al [10], and spin splitting has been investigated theoretically in a one dimensional quantum ring with three leads in the presence of the Rashba spin–orbit interaction by Földi et al., using the wave guide theory [11]. Recently, logical gate responses of mesoscopic rings connected symmetrically to two external leads in the presence of the Aharonov-Bohm (AB) magnetic flux and external gate voltages have been studied; however, the results are not suitable for spintronic circuits since the total and not spin resolved conductance has been investigated. Also, AB flux penetrated the ring plays a crucial role for whole logic gate operations [12].

In this paper, we use the Landauer framework of ballistic transport and demonstrate that the spin perfect polarized current can be controlled by the Rashba and Dresselhaus spin-orbit interaction and the magnetic flux \( \varphi \) [Aharonov-Bohm flux (AB)]. This magnetic flux add spin-dependent geometric phases to the electron wave function in the ring and lead to the so-called Aharonov-Casher (AC) effect [13], which is the relativistic cousin of the AB effect. In the AB effect [14], the electron wave function acquires a geometric phase after traveling the ring threaded by an AB magnetic flux. P Spin sensitive quantum interferences under the influence of the AB and AC effects make the quantum rings to have a practical significance for designing nano-electromagnetic spin devices, such as spin switches, detectors, spin transistors, filters [15–20], and scalable devices for quantum information processing. So, the scattering theory has been used to calculate the spin transmission. In this paper, both of the RSOI and DSOI exist simultaneously and behave like in-plane momentum-dependent effective magnetic fields. So, we consider a mesoscopic ring with three-leads formed in terms of both the Rashba and the Dresselhaus SOIs, the AB flux, and the electron energy, at a two-dimensional electrons gas, and try to present the full spin polarization
criterion for it. It is shown that this system can act as a perfect spin filter only in the presence of both of RSOI and DSOI. We calculated spin depended transmission coefficients analytically and the effects of coupling between the leads and rings on spin transport properties are taken into account. The scheme of the paper is as follows. In Sec. II, a theoretical model is presented. The eigenvalue problem in the presence of the Rashba and Dresselhaus spin–orbit interaction is solved analytically and the spin-dependent transmission coefficients are obtained. In Sec. III, the numerical results based on the equations derived in Sec. II are presented and discussed. The conditions that result in perfect spin polarization are determined in detail and the effects of each system parameter such as the electron energy and the magnetic flux on the spin polarization are investigated. Finally, a summary is given in Sec. IV.

**Theoretical Model**

We consider a mesoscopic ring with three leads as shown in Fig. 1. The system is mapped onto a one-dimensional virtual lattice. The left lead acts a source (S) and the two right leads act as drains (d₁ and d₂). In the tight-binding model, an electron can jump from one site to the nearest neighbor sites with a hopping energy matrix \([t]\).

![Fig. 1: A schematic view of a three-terminal semiconductor mesoscopic ring](image)

The total Hamiltonian for the system can be written as:

\[
H = \sum_p H_p + \sum_q H_{cq}
\]

where, \(H_p\) with \(p = c, s, d_1, d_2\) is the Hamiltonian of isolated ring, source leaded, up drain lead, and down lead, respectively, \(H_{cq}\) is the sum of contrib. form the ring, \(H_{c_1}\) and \(H_{c_2}\) with \(q = s, d_1, d_2\) is the Hamiltonian due to the coupling between the ring and the source lead, up drain lead, and down drain lead, respectively. In this paper, we consider source and drains electrodes consist of the same materials.

In the absence of the electron-electron and electron-phonon interaction, \(H_p\) and \(H_{cq}\) can be written as follow:

\[
H_p = -t_p \sum_{i=1}^{3N} (C_i \sigma_0 C_i^\dagger + C_i^\dagger \sigma_0 C_i) - t e^{i\alpha} C_{i+1} C_i^\dagger e^{-i\alpha} C_i^\dagger
\]

\[
H_{cq} = -t_{c_{pq}} \sigma_0 C_i + h.c.
\]

where, \(t_p\) is a matrix \(2 \times 2\) identity matrix in the spin space, Anderson-like on site disorder energy strength \(\varepsilon_i\). In the two component operator, \(C_i^\dagger = (c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger)\), \(c_{i\uparrow}^\dagger\) is the creation operator (annihilates) of an electron on the site \(i\) with spin-state. The RSOI and DSOI strength \(t^R\) and \(t^D\), respectively, and \([t]\) is a matrix \(2 \times 2\), which is given by \([t]\):
We consider 3N atomic sites on the ring (Fig. 1), \(\sigma_x\) and \(\sigma_y\) are the Pauli matrices. The Peierls phase factor \(e^{i\pi x}\) with \((2\pi/3N)\sigma_0/\sigma_0\) describes the influence of the magnetic field in terms of the magnetic flux \(\Phi\), threaded by the ring, and \(\sigma_0 = \hbar c/\varepsilon\) is the flux quantum.

In, is periodic boundary condition imposed by \(3N + 1 = 1\). We assume source and drains electrodes consist of the same materials. We consider linear transport regime conductance \(G\) of the interferometer can be obtained using the Landauer conductance formula [23, 24]:

\[
G = \frac{2e^2}{h} T_{pq}^{\sigma\sigma},
\]

where \(T_{pq}^{\sigma\sigma}\) is the transmission probability of an electron across it can be expressed in terms of Green’s function and its coupling to the side-attached electrons by [25]:

\[
T_{pq}^{\sigma\sigma} = Tr \left[ \Gamma_d^p G_c \Gamma_d^q G_c^* \right]
\]

where \(G_c\) and \(G_c^*\) are respectively the retarded and advanced Green’s function. Here \(\Gamma_d^p\) and \(\Gamma_d^q\) describe the coupling of the source and drains \((d = d_1 + d_2)\). The coherent transport is then calculated using the Landauer’s formula and the retarded Green’s function of the ring is computed as:

\[
G_c = \left( E^+ - H_c - \sum_p \Sigma_p \right)^{-1}
\]

where \(E^+\) is complex energy for one way to incorporate the boundary into the equation and self-energy \(\sum_p\) describes the effect of the leads on the conductor of \(\sum_p\) with \(p = s,d_1,d_2\) being the self energy of the source and drains electrodes. The \(\Sigma_p\) calculated as \(\sum_p t_p^* g_p t_p\), where \(t_p = -t_0 \sigma_0\) and \(g_p = -1/t_p e^{ik_0 \sigma_0}\), where \(t_p\) consists of ring-lead interaction and the \(g_p\) is the surface Green’s function of the electrodes and \(k_0\) satisfies the relation \(E = -2t_p \cos(k_0 \alpha)\). Therefore, the conductance of the ring is obtained as the Landauer equation.

**Results and Discussion**

We consider different conditions and several cases in numerical calculation that make our system a perfect spin polarization. Before going into discussion, let us use first assign the values of different parameters which are used for our numerical results. We will present results for a ring described by \(N = 110\) sites and in the strong coupling regime where the strong of ring-to-coupling, \(t\), is comparable to the hopping strength, \(t\), in the system (we take \(t = 1\) as the energy scale). We suppose that the incoming electrons can be sum of spin-up and spin-down with equal amplitude and the geometry leads is a symmetric geometry i.e. \(\varphi_1 = \varphi_2 = \varphi_3\). Therefore, the geometry can be defined as \(T_1^+ = T_2^+\) and \(T_1^- = T_2^-\).

Firstly, we analyzed the transport as function of the electron Fermi energy and SO interaction. Fig. 2 shows the spin-up and spin-down transmission probabilities as a function of the electron Fermi energy for fixed non-vanishing values of RSOI \((t^R = 0.135)\) and the values of DSOIs. In the presence of the half-integer values of DSOI (the first and last panel) there is a little difference between spin-up and spin-down conductance. This is due to the SOI-induced phase shift which leads to the complete destructive interference between outgoing beams coming from the lower and upper out leads of the ring. We examined that this is also true for others values of DSOIs. By increasing the strength of DSOI in Fig. 2, from top to the third panel, the degeneracy between spin-up and spin-down transmission is removed and there will be a modulation in the changes between spin-up and spin-down conductance. Therefore, the resulting spin polarization will increase. By the third to last panel with increasing the strength DSOI, the amplitudes of spin-up and spin-down transmission probability decrease and their degeneration increases. Therefore, by increasing the strength of DSOI, the spin polarization in which this goes to zero.
Also, we can see in third panel that perfect spin polarization occurs in values of \( t^D = 0.225 \). The maximums and minimums of \( T^\uparrow \) and \( T^\downarrow \) occurs at \( \bar{E} = 0 \) and \( \bar{E} = -0.192 \& 0.204 \), respectively. In such point where \( T^\uparrow = T^\downarrow = 1 \) with filter spin in the absence of flux magnetic through the ring and in zero conductance energy. There is not only one value of SOI strengths that results perfect spin polarization. In order to obtain the aforementioned specific values of DSOI and RSOI strength, we presented in Fig. 3, the contour map of spin polarization plotted in \( t^R - t^D \) plane for \( \bar{E} = 0.01 \) [the first panel], \( \bar{E} = 0.18 \) [the second panel] and \( \bar{E} = 0.22 \) [the third panel]. It can be observed that the system under consideration has nonzero spin polarization only when difference between \( t^R \) and \( t^D \) is small.

The spin polarization oscillates quasi periodically as the strength of the RSOI and DSOI increases, and the spin polarization occurs only in regions near the bisector of \( t^R - t^D \) plane, since the Hamiltonian of the RSOI and DSOI are equivalent. In all the panels we observe perfect spin polarization at specific values equal to \( (t^R - t^D) \), these specific values are \( (0.225; 0.14) \), \( (0.235; 0.13) \) and \( (0.215; 0.17) \) for all panels, respectively. In this case, as it can be seen from Fig. 3, the spin polarization at \( E=0.22 \) greater area than other energy values. This increase is due to the increased constructive interaction between the RSOI and DSOI in the amount of energy. The lower panels in Fig. 3, show a close up of the upper panels in the area inside the black rectangle, and show that the spin polarization reach unity at this specific values. Fig. 3, also shows that all panels of a system under consideration has non-zero spin polarization only when difference between \( t^R \) and \( t^D \) is small. This feature provides a possible way to detect the strength of the DSOI since the strength of the RSOI can be tuned by the external electric fields and it is known that the spin polarization is an odd function under exchanged of the SOIs strengths; i.e. \( P(t^R, t^D, E) = -P(t^D, t^R, E) \).
In the absence of magnetic flux, the presence of both RSOI and DSOI is necessary in order to obtain symmetric tree-terminal ring acts as a perfect spin selective device. However, verifying this conclusion needs a spacious search in \(\{t_R, t_D, E\}\) parameter space. We search the specific values of \(t_R\) and \(t_D\) for which perfect spin filtering conditions are established. In Fig. 4, we show how the spin polarization varies with the strength of the RSOI and the electron energy in presence of various values of DSOI strength. The RSOI behaves like an effective in-plane momentum-dependent magnetic field. The fully spin polarized in incoming the lead will be changed to the spin down in the outgoing lead \((d_1)\) at large strength SOIs. For \(t_D=0\) [first panel], the maximums of spin polarization is 0.76 occur at \(t_R=0.025\) and some periodic values of the Fermi energy. Therefore, for SOI strengths equal to 0.16, there are many regions other than values of SOI strengths in \(E-t_R\) or \(E-t_D\) plane, in which the incoming electron with spin state down and vice versa. In this case, the system acts as a filter spin. By increasing \(t_D\) the extremum peaks of the spin polarization shift to larger values of \(t_R\). At the special values \(t_D=0.16\) (seventh panel), we have a unit spin polarization for \(t_R=0.22\) and some periodic values of the Fermi energy. Lake of perfect spin polarization in these values can be interpreted as follows. When a mesoscopic ring bridges asymmetrically tree breads, the original cylindrical symmetric of the ring breaks due to the interplay between RSOI and DSOI which introduces a periodic potential the ring and consequently leads to an anisotropic spin transport.

So far, we have studied perfect polarization for \(\varphi_1=\varphi_2=\varphi_3=120^\circ\) with different values of Dresselhaus and Rashba constant. Now we see the other values of \(\varphi_3\), still with the symmetric geometry, that lead to the perfect spin polarization. Fig. 5 shows the transmission coefficient and spin polarization as function of the electron Fermi energy for various values of \(\varphi_3\). It can be seen from Fig. 5 that except for the case of \(\varphi_3=90,120^\circ\), other values of \(\varphi_3\) that result into the perfect polarization, depend on the magnetic flux. In the first and second panels for values \(\varphi_3=90,120\) the peak of spin polarization occurs at \(E=-0.156,0.276\) and \(E=-0.8,0.8\) respectively. But in the other values of \(\varphi_3=180,270^\circ\) perfect spin polarization not exist for any of \(t_R\) and \(t_D\). Therefore, to achieve perfect spin polarization in this value
angels of leads $d_1$ and $d_2$, should applied magnetic flux AB (Aharonov–Bohm phase) through the ring. This is due to the magnetic flux-induced phase shift which leads to the complete destructive interference between the outgoing beams coming from the lower and upper leads of the ring in the spin-down channel. So, when a mesoscopic ring bridges asymmetrically three leads, the original cylindrical symmetry of the ring breaks due to the interplay between RSOI and DSOI which introduces a periodic potential along the ring and consequently leads to an anisotropic spin transport [26], i.e., the conductance is sensitive to the positions of the incoming and outgoing leads. Therefore, perfect spin polarization will not occur in the all of the angle of leads in the ring asymmetrically coupled to the source.

FIG. 4: (Color online) Contour plot of spin polarization in $E$–$t_R$ plane for various values of DSOI strength $t_D$. 

$t_R=0.435$
$t_D=0.485$

$t_R=0.135$
$t_D=0.225$
Conclusion

We have investigated the spin transport for a quantum ring with three leads in the presence of the Rashba and Dresselhaus spin–orbit effect using the use of the Landauer formalism. Our analysis focused on the effect of both the RSOI and DSOI simultaneously which is the case in practice. Indeed, in a clean AB ring coupled symmetrically to reservoirs, perfect spin polarization doesn’t occur in the presence of only one type of SOIs. We showed that total spin filtering with zero reflection is possible for strongly coupled leads with values species SOIs and $\varphi_3$ without the magnetic flux. In the case of symmetric geometry, to obtain perfect spin polarization, the contour maps of spin polarization are plotted in terms of the normalized energy, Rashba and Dresselhaus constant. Finally, we have also obtained the optimum values of the Rashba and Dresselhaus constant for perfect spin polarization in the case of a special geometry for values $\varphi_3=90, 120, 180, 270$ and again the conditions that lead to the perfect spin polarization have been specified.

References:

УДК 537.311.33

**Влияние эффекта спин-орбитального взаимодействия Рашбы и Дрессельхауза на квантовый транспорт и спиновую фильтрацию в трехтерминальном квантовом кольце**

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**Аннотация.** Изучена спиновая проводимость в квантовом кольце с одним входом и двумя выходами в присутствии спин-орбитального взаимодействия Рашбы (RSOI) и спин-орбитального взаимодействия Дрессельхауза (DSOI). Также изучены условия, необходимые для идеальной спиновой поляризации, в том числе значения напряженности и силы связи при эффекте Рашбы и Дрессельхауза и значения энергии электрона. Наши расчеты выполнены с использованием неравновесной функции Грина и граничного условия Гриффита в рамках модели сильной связи. Показано, что спин-поляризованным транспортом и поляризуемостью можно управлять с помощью RSOI и/или DSOI при воздействии эффекта приложенного магнитного поля на квантовое кольцо, а также при смещении, прикладываемом между входными и выходными выводами. Результаты этой работы могут быть использованы при разработке совершенных спин-инверторов на основе современных полупроводниковых технологий. Также исследованы эффекты относительных положений стоковых электродов на спиновую поляризацию.

**Ключевые слова:** трехтерминальное квантовое кольцо; эффекты Рашбы и Дрессельхауза.